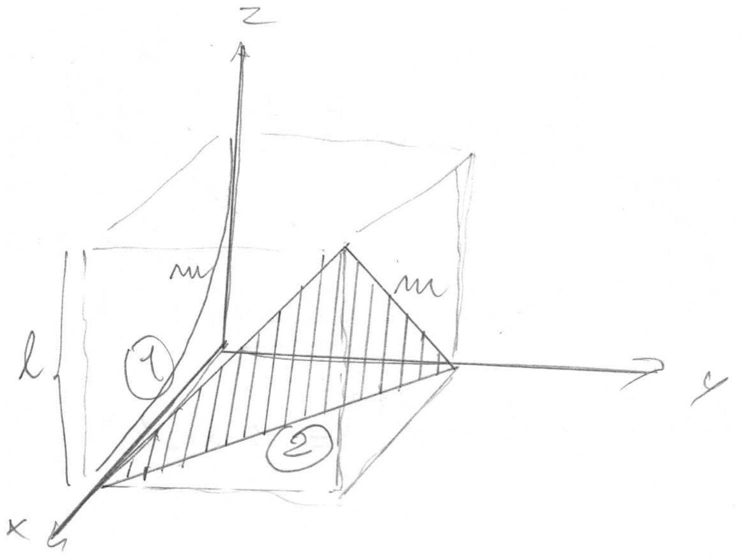
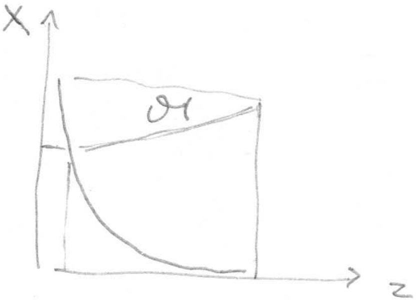


(a) MATRICE D'INERZIA
(b) MOMENTI E ASSI PRINCIPALI

1



$$\rho = \frac{m}{\pi/2}, \quad dm = \rho\, d\theta$$

$$x = l(1-\operatorname{sen}\theta)$$

$$z = l(1-\cos\theta)$$

$$I_x = \int z^2\, dm = \frac{2m}{\pi} l^2 \int_0^{\pi/2} (1-\cos\theta)^2\, d\theta$$

$$= \frac{2m}{\pi} l^2 \int_0^{\pi/2} (1+\cos^2\theta - 2\cos\theta)\, d\theta$$

$$= \frac{2ml^2}{\pi} \left\{ \frac{\pi}{2} + \frac{\pi}{4} - 2\operatorname{sen}\theta \Big|_0^{\pi/2} \right\} = \frac{2ml^2}{\pi} \left\{ \frac{3\pi}{4} - 2 \right\}$$

$$= \frac{ml^2}{2} \left\{ 3 - 8/\pi \right\} = I_z \Rightarrow I_y = ml^2(3-8/\pi)$$

$$I_{xz} = -\int xz\, dm = -\frac{2ml^2}{\pi} \int_0^{\pi/2} (1-\cos\theta)(1-\operatorname{sen}\theta)\, d\theta$$

$$= -\frac{2ml^2}{\pi} \int_0^{\pi/2} (1+\cos\theta\operatorname{sen}\theta - \cos\theta - \operatorname{sen}\theta)\, d\theta$$

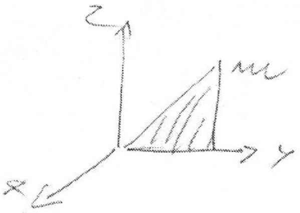
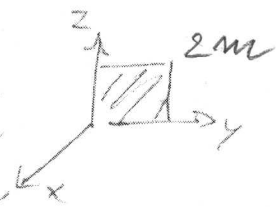
$$= -\frac{2ml^2}{\pi} \left\{ \frac{\pi}{2} + \frac{\operatorname{sen}^2\theta}{2}\Big|_0^{\pi/2} - \operatorname{sen}\theta\Big|_0^{\pi/2} + \cos\theta\Big|_0^{\pi/2} \right\}$$

$$= -\frac{2ml^2}{\pi} \left\{ \frac{\pi}{2} + \frac{1}{2} - 1 - 1 \right\} = -\frac{2ml^2}{\pi} \left\{ \frac{\pi}{2} - \frac{3}{2} \right\} = -ml^2\left(1-\frac{3}{\pi}\right)$$

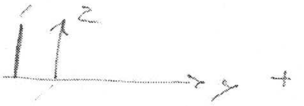
$$I_{xy} = I_{yz} = 0$$

$$I^{(1)} = ml^2 \begin{pmatrix} \frac{1}{2}\left(3-\frac{8}{\pi}\right) & 0 & -(1-3/\pi) \\ 0 & (3-8/\pi) & 0 \\ -(1-3/\pi) & 0 & \frac{1}{2}\left(3-8/\pi\right) \end{pmatrix}$$

②

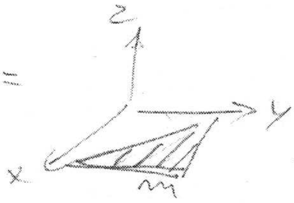
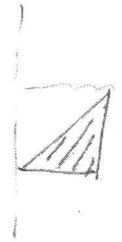
PER PROIEZIONE $I_x^{(2)} =$  $= \frac{1}{2}$ 

SISTEMI PIANI

$= \frac{1}{2}\Big($  $+$  $\Big)$

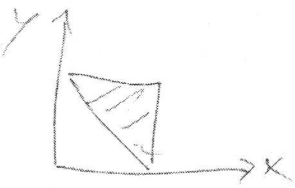
$= \frac{1}{2}\left(\frac{1}{3}(2m)l^2 + \frac{1}{3}(2m)l^2\right) = \frac{2}{3} m l^2$

SIMILMENTE $I_y^{(2)} = I_x^{(2)}$

PER PROIEZIONE $I_z^{(2)} =$  $= 2\Big($  $\Big)$

$= m l^2$

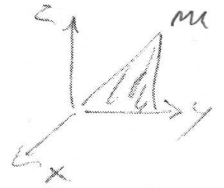
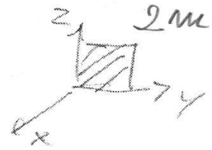
IN PIANO



$I_y = \int x^2\, dm = \frac{m}{l^2/2}\int_0^l dx \int_{l-x}^{l} dy\, x^2$

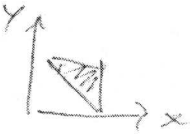
$= \frac{2m}{l^2}\int_0^l dx\, x^3 = \frac{2m}{l^2}\frac{l^4}{4} = \frac{1}{2} m l^2$

PER PROIEZIONE

$I_{yz}^{(2)} =$  $= \frac{1}{2}$  $\overset{\text{HUYGENS-STEINER}}{=} \frac{1}{2}\left(-2m\left(\frac{l}{2}\right)^2\right) = -\frac{1}{4} m l^2$

$\parallel$

$I_{xz}^{(2)}$

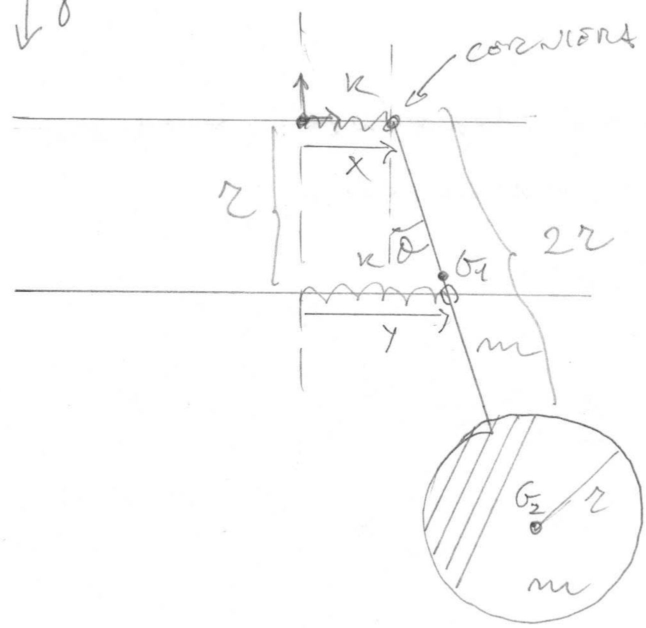
$I_{yx}^{(2)} =$  $= -\int xy\, dm = -\frac{2m}{l^2}\int_0^l dx \int_{l-x}^{l} dy\, xy$

$= -\frac{2m}{l^2}\int_0^l dx\, \frac{x}{2}\left(l^2-(l-x)^2\right) = -\frac{2m}{l^2}\int_0^l dx\, \frac{x}{2}\left(-x^2+2xl\right)$

$= -\frac{2m}{l^2}\left(-\frac{l^4}{8}+\frac{l^4}{3}\right) = -\frac{5}{12} m l^2$

$$I^{(2)} = m\,l^2 \begin{pmatrix} 2/3 & -5/12 & -1/4 \\ -5/12 & 1/3 & -1/4 \\ -1/4 & -1/4 & 1 \end{pmatrix}$$

$kr = mg$



(a) CALCOLARE V
PUNTI STAZIONARI
STABILITÀ
(b) CALCOLARE T
(c) MODI E PULSAZIONI

$$r\,\mathrm{tg}\,\theta = y - x \Rightarrow \theta = \mathrm{arctg}\,\frac{y-x}{r}$$

$$\Rightarrow \dot\theta = \frac{1}{1+\left(\frac{y-x}{r}\right)^2}\,\frac{\dot y - \dot x}{r}$$

$$G_1 = (x + r\sin\theta,\ -r\cos\theta)$$

$$G_2 = (x + 3r\sin\theta,\ -3r\cos\theta)$$

$$V = \frac{1}{2}k(x^2+y^2) - mgr(\cos\theta + 3\cos\theta)$$

$$= \frac{1}{2}k(x^2+y^2) - 4kr^2\cos\theta$$

TAYLOR A (0, 0)

$$\cos\theta = \frac{1}{\sqrt{1+\mathrm{tg}^2\theta}} = \frac{1}{\sqrt{1+\left(\frac{y-x}{r}\right)^2}} \simeq 1 - \frac{1}{2}\left(\frac{y-x}{r}\right)^2 + \dots$$

$$V = \frac{1}{2}k(x^2+y^2) - 4kr^2\frac{1}{\sqrt{1+\left(\frac{y-x}{r}\right)^2}}$$

$$\frac{\partial V}{\partial x} = kx + 2kr^2\frac{1}{\left(1+\left(\frac{y-x}{r}\right)^2\right)^{3/2}}\left(-2\frac{(y-x)}{r}\right)$$

$$\frac{\partial V}{\partial y} = ky + 2kr^2\frac{1}{\left(1+\left(\frac{y-x}{r}\right)^2\right)^{3/2}}\left(+2\frac{(y-x)}{r}\right)$$

$$\frac{\partial V}{\partial x} = \frac{\partial V}{\partial y} = 0$$

PRENDENDO DIFFERENZA E SOMMA $y - x = 0$, $x + y = 0$

QUINDI $x = y = 0$ È IL PUNTO STAZIONARIO.

L'ESPANSIONE QUADRATICA DI V È

$$V = \frac{1}{2}k(x^2+y^2) - 4kr^2\left\{1 - \frac{1}{2}\left(\frac{y-x}{r}\right)^2\right\}$$

$$= \frac{1}{2}k\left(3x^2+3y^2-4xy\right) + \text{cost}$$

$$\Rightarrow \quad B = \begin{pmatrix} 3 & -2 \\ -2 & 3 \end{pmatrix} \Rightarrow \text{DEF POSITIVA} \Rightarrow \text{STABILITÀ}$$

$$\dot{G}_1 = \left(\dot{x} + r\cos\theta\,\dot{\theta},\ r\sin\theta\,\dot{\theta}\right)$$

$$\dot{G}_2 = \left(\dot{x} + 3r\cos\theta\,\dot{\theta},\ 3r\sin\theta\,\dot{\theta}\right)$$

$$T = \frac{1}{2}m\left(\dot{G}_1^2 + \dot{G}_2^2\right) + \frac{1}{2}\left(\frac{1}{12}m(2r)^2 + \frac{1}{2}mr^2\right)\dot{\theta}^2$$

$$= \frac{1}{2}m\left\{\dot{x}^2 + r^2\dot{\theta}^2 + 2r\cos\theta\,\dot{x}\dot{\theta} + \dot{x}^2 + 9r^2\dot{\theta}^2 + 6r\cos\theta\,\dot{x}\dot{\theta}\right\}$$
$$+ \frac{5}{12}mr^2\dot{\theta}^2$$

$$= \frac{1}{2}m\left\{2\dot{x}^2 + \frac{65}{6}r^2\dot{\theta}^2 + 8r\cos\theta\,\dot{x}\dot{\theta}\right\}$$

$$= \frac{1}{2}m\left\{2\dot{x}^2 + \frac{65}{6}r^2\left[\frac{(\dot{y}-\dot{x})/r}{1+\left(\frac{y-x}{r}\right)^2}\right]^2 + 8r\dot{x}\,\frac{\dot{y}-\dot{x}}{r}\left(1+\left(\frac{y-x}{r}\right)^2\right)^{-3/2}\right\}$$

$$\simeq \frac{1}{2}m\left\{2\dot{x}^2 + \frac{65}{6}\left(\dot{y}^2+\dot{x}^2-2\dot{y}\dot{x}\right) + 8\dot{x}\dot{y} - 8\dot{x}^2\right\}$$

$$\simeq \frac{1}{2}m\left\{\frac{29}{6}\dot{x}^2 + \frac{65}{6}\dot{y}^2 - \frac{41}{3}\dot{x}\dot{y}\right\}$$

$$A = \frac{m}{6}\begin{pmatrix} 29 & -41 \\ -41 & 65 \end{pmatrix}$$