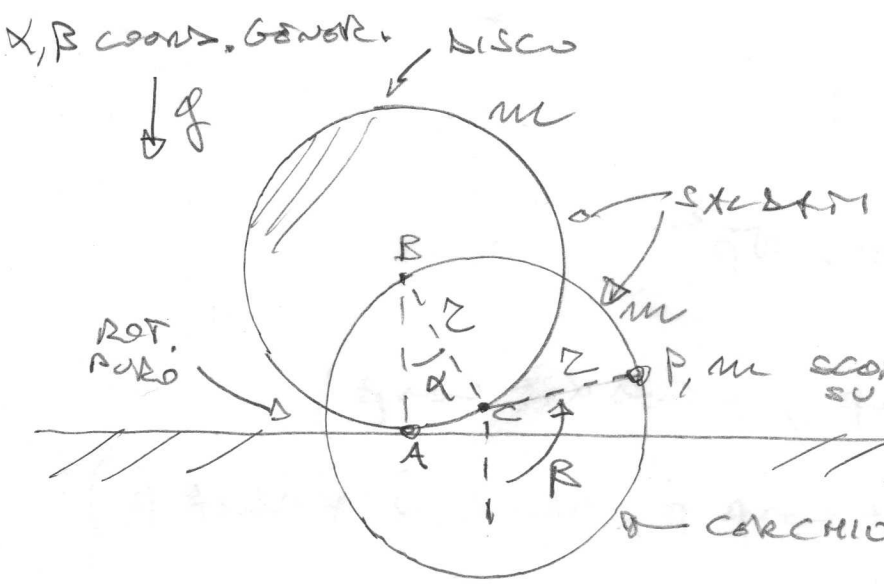
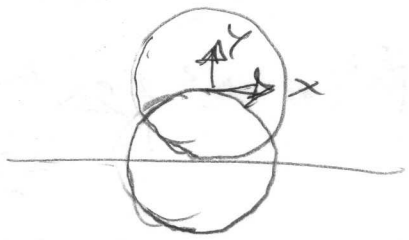


1) scrivere V, punti staz. stabilità
2) scrivere T
3) piccole oscillaz.



$$R = (-r\alpha, 0)$$

$$C = (-r\alpha + r\sin\alpha, -r\cos\alpha)$$

$$P = (-r\alpha + r\sin\alpha + r\sin\beta, -r\cos\alpha - r\cos\beta)$$

$$I_A^{\text{DISCO}} = \frac{1}{2}mr^2 + mr^2 = \frac{3}{2}mr^2$$

$$I_A^{\text{CERCHIO}} = mr^2 + m\left(2r\sin\frac{\alpha}{2}\right)^2$$

$$I_A = I_A^{\text{DISCO}} + I_A^{\text{CERCHIO}}$$

$$V = -mgr\cos\alpha - mgr\cos\alpha - mgr\cos\beta$$
$$= -mgr(2\cos\alpha + \cos\beta)$$

$$0 = \frac{\partial V}{\partial\alpha} = 2mgr\sin\alpha \quad \begin{cases} (\alpha,\beta) \text{ con:} \\ \alpha = 0, \pi, \dots \\ \beta = 0, \pi \end{cases}$$

$$0 = \frac{\partial V}{\partial\beta} = mgr\sin\beta$$

sono punti staz.

$$\frac{\partial^2 V}{\partial\alpha^2} = 2mgr\cos\alpha$$

$$\frac{\partial^2 V}{\partial\beta^2} = mgr\cos\beta$$

$$\frac{\partial^2 V}{\partial\alpha\partial\beta} = 0$$

$$\Rightarrow B = mgr\begin{pmatrix} 2 & 0 \\ 0 & 1 \end{pmatrix}$$

per $\alpha = 2\pi n$, $\beta = 0$

altrimenti instabile

$\omega_{DISCO} = \omega_{CERCHIO} = \dot{\alpha}$

$$T = \frac{1}{2} I_A \omega^2 + \frac{1}{2} m v_P^2$$

$$P = r(-\alpha + \sin\alpha + \sin\beta,\ -\cos\alpha - \cos\beta)$$

$$\dot{P} = r(-\dot{\alpha} + \cos\alpha\,\dot{\alpha} + \cos\beta\,\dot{\beta},\ \sin\alpha\,\dot{\alpha} + \sin\beta\,\dot{\beta})$$

$$\dot{P}^2 = r^2 \{ 2\dot{\alpha}^2 + \dot{\beta}^2 - 2\dot{\alpha}^2\cos\alpha - 2\dot{\alpha}\dot{\beta}\cos\beta + 2\dot{\alpha}\dot{\beta}[\cos\alpha\cos\beta + \sin\alpha\sin\beta] \}$$

$$= r^2 \{ 2\dot{\alpha}^2(1-\cos\alpha) + \dot{\beta}^2 + 2\dot{\alpha}\dot{\beta}[\cos\alpha\cos\beta + \sin\alpha\sin\beta - \cos\beta] \}$$

$$T = \frac{1}{2} m r^2 \left(\frac{5}{2} + 4\sin^2\frac{\alpha}{2}\right)\dot{\alpha}^2 + \frac{1}{2} m r^2 \{ 2\dot{\alpha}^2(1-\cos\alpha) + \dot{\beta}^2 + 2\dot{\alpha}\dot{\beta}[\cos\alpha\cos\beta + \sin\alpha\sin\beta - \cos\beta] \}$$

$$A = m r^2 \begin{pmatrix} 5/2 & 0 \\ 0 & 1 \end{pmatrix}$$

QUINDI I MODI SONO GIÀ DISACCOPPIATI CON PULSAZIONI

$$\omega_1 = \sqrt{\frac{4}{5}\frac{g}{r}}, \qquad \omega_2 = \sqrt{\frac{g}{r}}$$