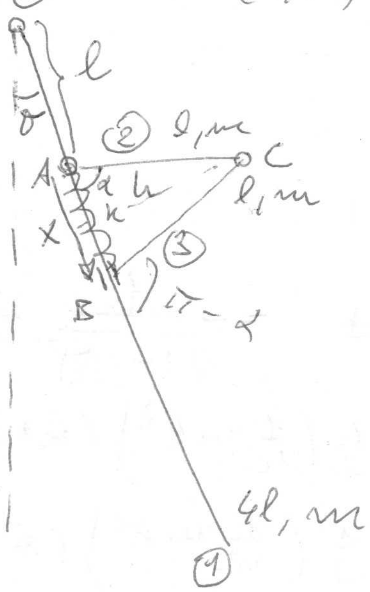

(x, θ) coord. gen.

$kl = mg$

a) V, punti staz.

b) T

$$V = -mg\,2l\cos\theta - 2mg\left\{\left(l+\frac{x}{2}\right)\cos\theta - \frac{h}{2}\,\mathrm{sen}\,\theta\right\} + \frac{1}{2}kx^2$$

$$h = \sqrt{l^2 - \left(\frac{x}{2}\right)^2}$$

$$V = -2kl^2\cos\theta - 2kl\left\{\left(l+\frac{x}{2}\right)\cos\theta - \frac{1}{2}\sqrt{l^2 - x^2/4}\,\mathrm{sen}\,\theta\right\} + \frac{1}{2}kx^2$$

$$= -kl\left\{(4l+x)\cos\theta - \sqrt{l^2-x^2/4}\,\mathrm{sen}\,\theta\right\} + \frac{1}{2}kx^2$$

$$\frac{\partial V}{\partial x} = -2kl\left\{\frac{1}{2}\cos\theta + \frac{1}{8}\frac{x}{\sqrt{l^2-x^2/4}}\,\mathrm{sen}\,\theta\right\} + kx$$

$$\frac{\partial V}{\partial \theta} = 2kl^2\,\mathrm{sen}\,\theta - 2kl\left\{-\left(l+\frac{x}{2}\right)\mathrm{sen}\,\theta - \frac{1}{2}\sqrt{l^2-x^2/4}\cos\theta\right\}$$

$$\frac{\partial V}{\partial \theta} = 0 \Rightarrow 0 = \left(2l+\frac{x}{2}\right)\mathrm{sen}\,\theta + \frac{1}{2}\sqrt{l^2-x^2/4}\cos\theta$$

$$\Rightarrow \mathrm{tg}\,\theta = -\frac{\sqrt{l^2-x^2/4}}{4l+x} \qquad (*)$$

$$\cos\theta = \frac{1}{\sqrt{1+\mathrm{tg}^2\theta}} = \frac{4l+x}{\sqrt{(4l+x)^2+l^2-x^2/4}}$$

$$\mathrm{sen}\,\theta = \frac{\mathrm{tg}\,\theta}{\sqrt{1+\mathrm{tg}^2\theta}} = -\frac{\sqrt{l^2-x^2/4}}{\sqrt{(4l+x)^2+l^2-x^2/4}}$$

$\frac{\partial V}{\partial x} = 0 \Rightarrow x$ eqz. soddisfatta

$$-\ell\left\{\frac{4\ell+\frac{3}{4}x}{\sqrt{(4\ell+x)^2+\ell^2-x^2/4}}\right\}-x=0$$

Trovato x, θ si ricava da (*).

$$\ell\cos\alpha = x/2 \Rightarrow \alpha = \arccos\left(\frac{x}{2\ell}\right) \Rightarrow \dot\alpha = -\frac{1}{\sqrt{1-\left(\frac{x}{2\ell}\right)^2}}\frac{\dot x}{2\ell}$$

$$T=\frac{1}{2}\left(\frac{1}{3}m(4\ell)^2\right)\dot\theta^2+\frac{1}{2}m\left(\dot G_2^2+\dot G_3^2\right)+\frac{1}{2}\left(\frac{1}{12}m\ell^2\right)(\dot\theta+\dot\alpha)^2$$
$$+\frac{1}{2}\left(\frac{1}{12}m\ell^2\right)(\dot\theta-\dot\alpha)^2$$

$$G_2=\left(\ell\,\mathrm{sen}\,\theta+\frac{\ell}{2}\mathrm{sen}(\theta+\alpha),\ \ell\cos\theta+\frac{\ell}{2}\cos(\theta+\alpha)\right)$$

$$G_3=\left(\left(\ell+\frac{x}{2}\right)\mathrm{sen}\,\theta+\frac{\ell}{2}\mathrm{sen}(\theta+\alpha),\ \left(\ell+\frac{x}{2}\right)\cos\theta+\frac{\ell}{2}\cos(\theta+\alpha)\right)$$

$$\dot G_2=\ell\left(\cos\theta\,\dot\theta+\frac{1}{2}\cos(\theta+\alpha)(\dot\theta+\dot\alpha),\ -\mathrm{sen}\,\theta\,\dot\theta-\frac{1}{2}\mathrm{sen}(\theta+\alpha)(\dot\theta+\dot\alpha)\right)$$

$$\dot G_2^2=\ell^2\left\{\dot\theta^2+\frac{1}{4}(\dot\theta+\dot\alpha)^2+\dot\theta(\dot\theta+\dot\alpha)\left[\underbrace{\cos\theta\cos(\theta+\alpha)+\mathrm{sen}\,\theta\,\mathrm{sen}(\theta+\alpha)}_{\cos\alpha}\right]\right.$$
$$=\ell^2\left\{\dot\theta^2+\frac{1}{4}(\dot\theta+\dot\alpha)^2+\dot\theta(\dot\theta+\dot\alpha)\frac{x}{2\ell}\right\}$$

$$\dot G_3=\left(\frac{\dot x}{2}\mathrm{sen}\,\theta+\left(\ell+\frac{x}{2}\right)\cos\theta\,\dot\theta+\frac{\ell}{2}\cos(\theta+\alpha)(\dot\theta+\dot\alpha),\ \frac{\dot x}{2}\cos\theta-\left(\ell+\frac{x}{2}\right)\mathrm{sen}\,\theta\,\dot\theta\right.$$
$$\left.-\frac{\ell}{2}\mathrm{sen}(\theta+\alpha)(\dot\theta+\dot\alpha)\right)$$

$$\dot G_3^2=\frac{\dot x^2}{4}+\left(\ell+\frac{x}{2}\right)^2\dot\theta^2+\frac{\ell^2}{4}(\dot\theta+\dot\alpha)^2+\frac{\dot x\ell}{2}(\dot\theta+\dot\alpha)\left\{\underbrace{\mathrm{sen}\,\theta\cos(\theta+\alpha)-\cos\theta\,\mathrm{sen}(\theta+\alpha)}_{-\mathrm{sen}\,\alpha}\right.$$
$$+\ell\left(\ell+\frac{x}{2}\right)\dot\theta(\dot\theta+\dot\alpha)\left\{\underbrace{\cos\theta\cos(\theta+\alpha)+\mathrm{sen}\,\theta\,\mathrm{sen}(\theta+\alpha)}_{\cos\alpha}\right\}$$

con $\mathrm{sen}\,\alpha=\sqrt{1-\left(\frac{x}{2\ell}\right)^2}$