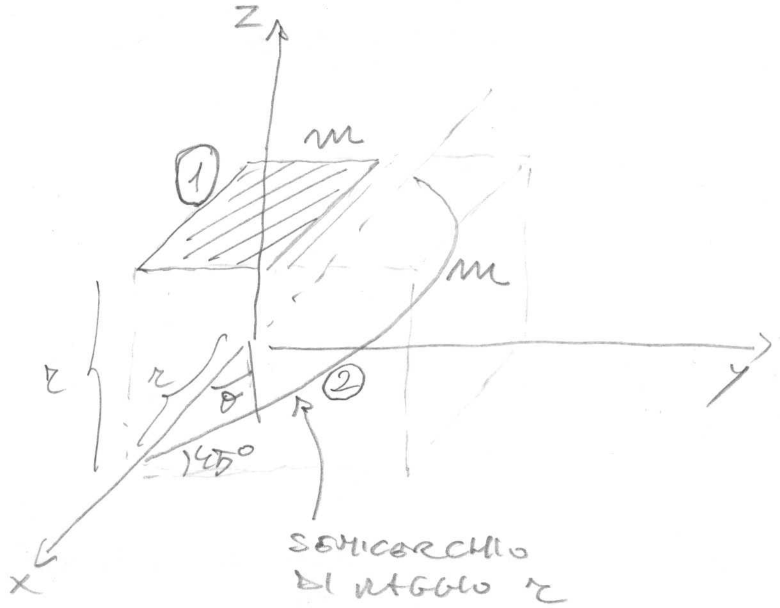


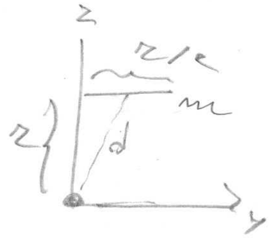
SEMICERCHIO DI RAGGIO r

CALCOLARE

I_x, I_y, I_{xy}

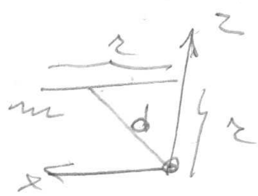
PER (1) E (2)

(1) PER CALCOLARE I_x SCHIACCIO LUNGO x



$$I_x = \frac{1}{12} m \left(\frac{r}{2}\right)^2 + m \left\{ \overbrace{r^2 + \left(\frac{r}{4}\right)^2}^{d^2} \right\} = \frac{13}{12} m r^2$$

PER CALCOLARE I_y SCHIACCIO LUNGO y



$$I_y = \frac{1}{12} m r^2 + m \left\{ \underbrace{r^2 + \left(\frac{r}{2}\right)^2}_{d^2} \right\} = \frac{4}{3} m r^2$$

PER CALCOLARE I_{xy} PROIETTO LUNGO z E USO HUYGENS-STEINER

$$I_{xy} = - m \frac{r}{4} \frac{r}{2} = - \frac{1}{8} m r^2$$

(2) INTRODUCO UN ANGOLO θ COME IN FIGURA, $\theta \in [0, \pi]$

$$x = r \cos\theta$$
$$y = \frac{1}{\sqrt{2}} r \sin\theta$$
$$z = \frac{1}{\sqrt{2}} r \sin\theta$$

$$dm = m \frac{d\theta}{\pi}$$

$$I_x = \int (y^2 + z^2)\, dm = \frac{r^2 m}{\pi} \int_0^{\pi} \sin^2\theta \, d\theta$$
$$= \frac{r^2 m}{2}$$

SI POTEVA TROVARLO CONSIDERANDO ASSE GIACENTE SU [illegible]

$$I_y = \int (x^2+z^2)\,dm = \frac{mr^2}{\pi}\int_0^{\pi}\left(\cos^2\theta + \frac{1}{2}\sin^2\theta\right) d\theta$$

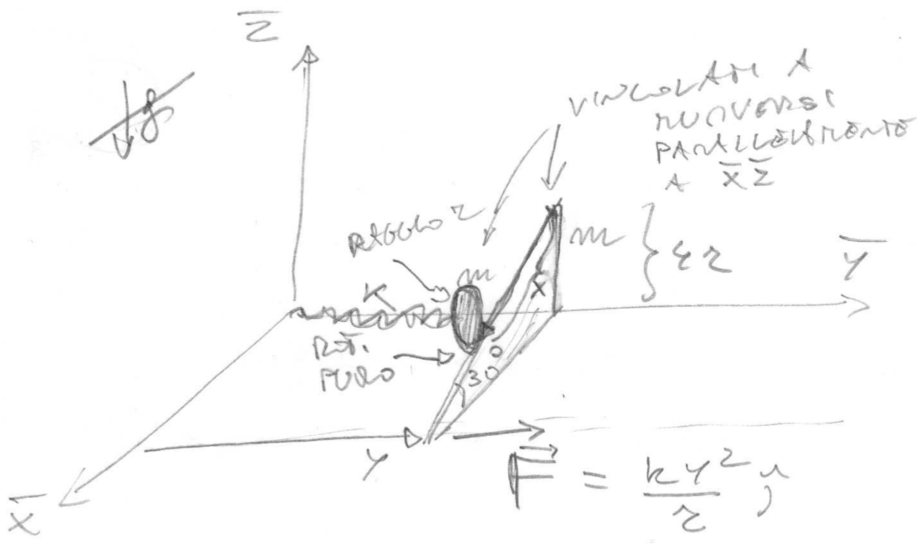
$$= \frac{mr^2}{\pi}\left\{\frac{\pi}{2} + \frac{\pi}{4}\right\} = \frac{3}{4}\,mr^2$$

$$I_{xy} = -\int xy\,dm = \frac{mr^2}{\sqrt{2}\,\pi}\int_0^{\pi}\cos\theta\,\sin\theta\,d\theta =$$

$$= \frac{mr^2}{\sqrt{2}\,\pi}\int_0^{0}\sin\theta\,d\sin\theta = 0$$

ANCHE PERCHÉ ZY È PIANO DI SIMMETRIA

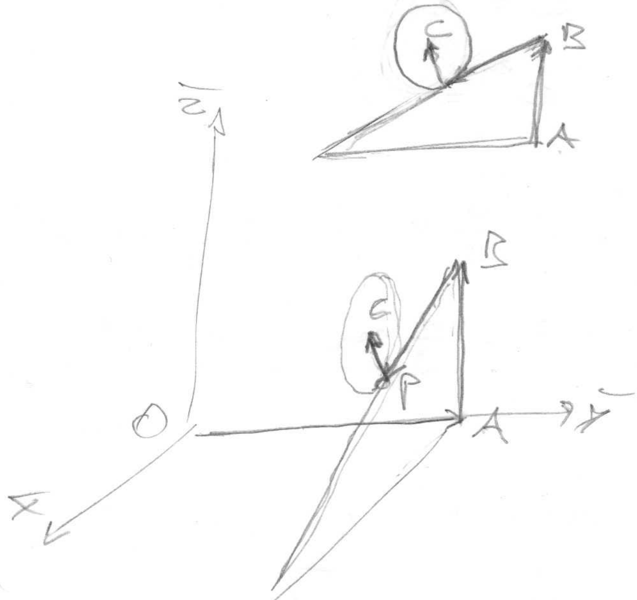
COORD. GENER: x, y



1) CALCOLARE V
PUNTI STAZ
STABILITA'

2) CALCOLARE T

3) PICCOLE OSCILL.



$$C-O = C-P+P-B+B-A+A-O$$

$$\vec{OC} = \vec{OA} + \vec{AB} + \vec{BP} + \vec{PC}$$

$$\vec{OA} = y\hat{j}$$

$$\vec{AB} = 4r\hat{k}$$

$$\vec{BP} = \frac{\sqrt{3}}{2}x\,\hat{i} - \frac{x}{2}\hat{k}$$

$$\vec{PC} = \frac{r}{2}\hat{i} + \frac{\sqrt{3}}{2}r\,\hat{k}$$

$$\vec{OC} = \left(\frac{r}{2} + \frac{\sqrt{3}}{2}x\right)\hat{i} + y\hat{j} + \left(4r + \frac{\sqrt{3}}{2}r - \frac{x}{2}\right)\hat{k}$$

$$\vec{OC}^2 = \left(\frac{r}{2} + \frac{\sqrt{3}}{2}x\right)^2 + y^2 + \left(4r + \frac{\sqrt{3}}{2}r - \frac{x}{2}\right)^2$$

$$V_F = -\int_0^y F\,dy = -\frac{k}{r}\int_0^y y^2\,dy = -\frac{ky^3}{3r}$$

$$V = \frac{1}{2}k\left\{\left(\frac{r}{2} + \frac{\sqrt{3}}{2}x\right)^2 + y^2 + \left(4r + \frac{\sqrt{3}}{2}r - \frac{x}{2}\right)^2\right\} - \frac{ky^3}{3r}$$

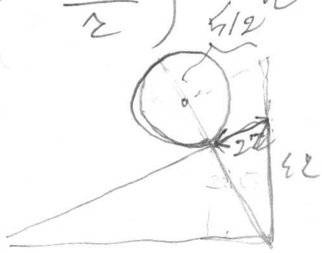
$$= \frac{1}{2}k\left\{x^2 + y^2 - 4rx\right\} - \frac{ky^3}{3r} + \text{cost.}$$

$$\frac{\partial V}{\partial x} = k\left\{x - 2r\right\}$$

$$\frac{\partial V}{\partial y} = k\left\{y - y^2/r\right\}$$

$\Rightarrow$ PUNTI STAZ: $x = 2r,\ y = 0$; $x = 2r,\ y = r$

$$\frac{\partial^2 V}{\partial x^2} = k \,, \quad \frac{\partial^2 V}{\partial x \partial y} = 0, \quad \frac{\partial^2 V}{\partial y^2} = k\left(1 - \frac{2y}{\ell}\right)$$



$$B = k\begin{pmatrix} 1 & 0 \\ 0 & 1 - \frac{2y}{\ell} \end{pmatrix}$$

QUINDI IL PRIMO PUNTO STAZ. E' STABILE CON $B = kI$, IL SECONDO E' INSTABILE $B = k\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}$

$$\dot{C} = \dot{\overrightarrow{OC}} = \frac{\sqrt{3}}{2}\dot{x}\,\hat{i} + \dot{y}\,\hat{j} - \frac{\dot{x}}{2}\,\hat{k}$$

$$\dot{C}^2 = \dot{x}^2 + \dot{y}^2$$

$$T = \frac{1}{2}m\dot{y}^2 + \frac{1}{2}m(\dot{x}^2 + \dot{y}^2) + \frac{1}{2}\left(\frac{1}{2}mr^2\right)\left(\frac{\dot{x}}{r}\right)^2$$

$$= \frac{1}{2}m\left\{\frac{3}{2}\dot{x}^2 + 2\dot{y}^2\right\} \Rightarrow A = m\begin{pmatrix} 3/2 & 0 \\ 0 & 2 \end{pmatrix}$$

QUINDI LE PULSAZIONI SONO $\omega_1 = \sqrt{\frac{2}{3}\frac{k}{m}}$ $\omega_2 = \sqrt{\frac{k}{2m}}$ E I MODI $v_1 = \begin{pmatrix} 1 \\ 0 \end{pmatrix}$ E $v_2 = \begin{pmatrix} 0 \\ 1 \end{pmatrix}$