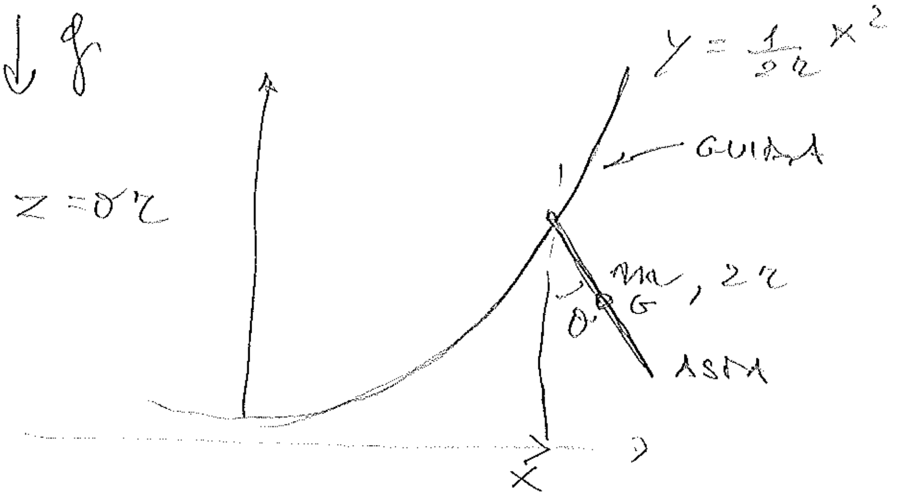
$\downarrow g$

$z = \theta r$



a) TROVARE V
PUNTI STAZIONARI
STABILITÀ

b) CALCOLARE T

c) PICCOLE OSCILLAZIONI (MODI E PULSAZ.)

$$G = \left(x + r\sin\theta ,\ \frac{1}{2r}x^2 - r\cos\theta \right)$$

$$\dot G = \left(\dot x + r\cos\theta\,\dot\theta ,\ \frac{x}{r}\dot x + r\sin\theta\,\dot\theta \right)$$

$$V = m g\, y_G = mg\left(\frac{1}{2r}x^2 - r\cos\theta\right)$$

$$\frac{\partial V}{\partial x} = \frac{mg}{r} x$$

$$\frac{\partial V}{\partial \theta} = mg r \sin\theta$$

$$\nabla V = 0 \iff x = 0 ,\ \theta = 0, \pi$$

$$\frac{\partial^2 V}{\partial x^2} = \frac{mg}{r} ,\quad \frac{\partial^2 V}{\partial \theta^2} = mg r\cos\theta$$

$\theta = 0$ STABILE, $\theta = \pi$ INSTABILE
DEFINISCO B USANDO LA COORDINATA Z AL POSTO DI X

$$B = \frac{mg}{r}\begin{pmatrix} 1 & 0 \\ 0 & 1 \end{pmatrix}$$

$$\dot G^2 = \dot x^2 + r^2\dot\theta^2 + 2\dot x r\cos\theta\,\dot\theta + 2x\dot x\sin\theta\,\dot\theta + \frac{x^2}{r^2}\dot x^2$$

$$T = \frac{1}{2} m \dot G^2 + \frac{1}{2}\left(\frac{1}{12} m (2r)^2\right)\dot\theta^2$$

$$= \frac{1}{2} m\left(\dot x^2 + r^2\dot\theta^2 + 2\dot x r\cos\theta\,\dot\theta + 2x\dot x\sin\theta\,\dot\theta\right) + \frac{1}{6} m r^2\dot\theta^2$$

$$\left.\frac{\partial^2 T}{\partial \dot{x}^2}\right|_{\substack{x=0\\ \theta=0}} = m \qquad \left.\frac{\partial^2 T}{\partial \dot{\theta}^2}\right|_{\substack{\theta=0\\ x=0}} = \frac{4}{3} m r^2$$

$$\left.\frac{\partial^2 T}{\partial \dot{x}\,\partial \dot{\theta}}\right|_{\substack{\theta=0\\ x=0}} = m r$$

$$A = m \begin{pmatrix} 1 & 1 \\ 1 & 4/3 \end{pmatrix}$$

$$\left(B - \omega^2 A\right) \vec{v} = 0$$

$$\omega^2 = \frac{g}{r} \lambda \qquad \left(\tilde{B} - \lambda \tilde{A}\right) \vec{v} = 0$$

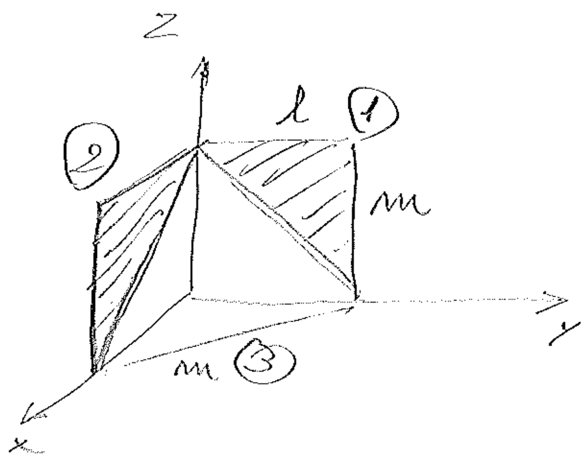
$$\det \begin{pmatrix} 1-\lambda & -\lambda \\ -\lambda & 1 - \frac{4}{3}\lambda \end{pmatrix} = 0$$

$$(1-\lambda)\left(1 - \frac{4}{3}\lambda\right) = \lambda^2$$

$$1 + \frac{4}{3}\lambda^2 - \frac{7}{3}\lambda = \lambda^2$$

$$\lambda^2 - 7\lambda + 3 = 0 \qquad \lambda = \frac{7 \pm \sqrt{49 - 12}}{2}$$

$$= \frac{7 \pm \sqrt{37}}{2}$$



a) MATRICE D'INERZIA

b) SIMMETRIE, MOMENTI E ASSI PRINCIPALI D'INERZIA

c) SE $m = 20\,g$, $l = 5\,cm$

$\bar{\omega} = (1,2,0)\ s^{-1}$

CALCOLARE L e T.

$$I_z^{(1)} = \int y^2\, dm = \rho \int_0^l dz \int_{l-z}^{l} y^2\, dy$$

$$= \frac{m}{l^2/2} \int_0^l dz\, \frac{1}{3} \left\{ l^3 - (l-z)^3 \right\} =$$

$$= \frac{2m}{3l^2} \left\{ l^4 + \frac{(l-z)^4}{4} \Big|_0^l \right\} = \frac{1}{2} m l^2$$

$$I_y^{(1)} = I_z^{(1)}$$

$$I_x^{(1)} = 2 I_z^{(1)} = m l^2$$

$$I_{yz}^{(1)} = -\int zy\, dm = -\rho \int_0^l z\, dz \int_{l-z}^{l} y\, dy$$

$$= -\frac{2m}{l^2} \int_0^l z\, dz\, \frac{1}{2} \underbrace{\left\{ l^2 - (l-z)^2 \right\}}_{2lz - z^2}$$

$$= -\frac{2m}{l^2} \left\{ \frac{l^4}{3} - \frac{l^4}{8} \right\} = -\frac{5}{12} m l^2$$

$$I_{xz}^{(1)} = I_{xy}^{(1)} = 0$$

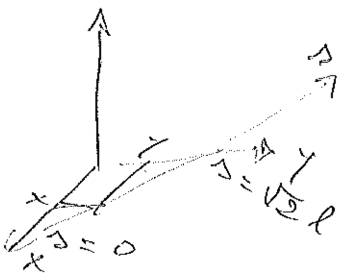
$$I^{(1)} = \frac{1}{12} m l^2 \begin{pmatrix} 12 & 0 & 0 \\ 0 & 6 & -5 \\ 0 & -5 & 6 \end{pmatrix}$$

$$I^{(2)} = \frac{1}{12} m \ell^2 \begin{pmatrix} 6 & 0 & -5 \\ 0 & 12 & 0 \\ -5 & 0 & 6 \end{pmatrix}$$

$$I_x^{(3)} = \frac{1}{3} m \ell^2 = I_y^{(3)}$$

$$I_z^{(3)} = 2 I_x^{(3)} = \frac{2}{3} m \ell^2$$

$$I_{zx}^{(3)} = I_{zy}^{(3)} = 0$$



$$I_{xy}^{(3)} = - \int x y \, dm = - \rho \int x y \, d\ell$$

$$= - \frac{m}{\sqrt{2}\ell} \int_0^{\sqrt{2}\ell} \left(\ell - \frac{s}{\sqrt{2}}\right) \frac{s}{\sqrt{2}} \, ds$$

$$= - \frac{m}{\sqrt{2}\ell} \left\{ \frac{\ell}{\sqrt{2}} \left(\frac{(\sqrt{2}\ell)^2}{2} \right) - \frac{1}{2} \frac{(\sqrt{2}\ell)^3}{3} \right\}$$

$$= - m \frac{\ell^2}{2} + m \frac{\ell^2}{3} = - \frac{1}{6} m \ell^2$$

$$I^{(3)} = \frac{1}{12} m \ell^2 \begin{pmatrix} 4 & -2 & 0 \\ -2 & 4 & 0 \\ 0 & 0 & 8 \end{pmatrix}$$

$$I = \frac{1}{12} m \ell^2 \underbrace{\begin{pmatrix} 22 & -2 & -5 \\ -2 & 22 & -5 \\ -5 & -5 & 20 \end{pmatrix}}_{M}$$

IL PIANO CHE CONTIENE L'ASSE z E BISETT. x-y

È DI SIMMETRIA QUINDI $\begin{pmatrix} 1 \\ -1 \\ 0 \end{pmatrix}$ È UN AUTOVETTORE

DI I ALL'AUTOVALORE

$$I_1 = \frac{1}{12} m l^2 \lambda_1 \quad \text{con} \quad \lambda_1 = 24 \text{ AUTOVALORE DI } M.$$

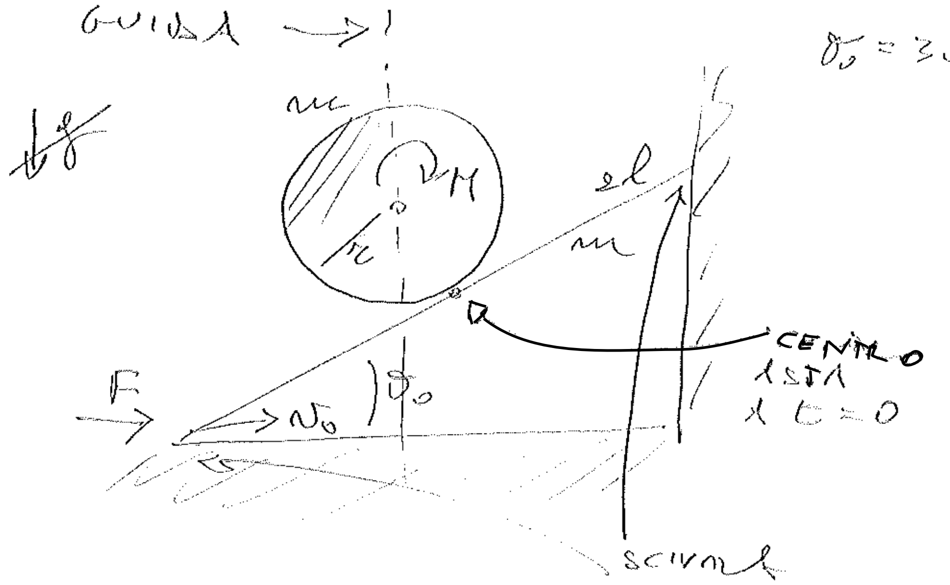
$$\det M = 8400 = \lambda_1 \lambda_2 \lambda_3 \Rightarrow \lambda_2 \lambda_3 = 350$$

$$\mathrm{Tr}\, M = 64 = \lambda_1 + \lambda_2 + \lambda_3 \Rightarrow \lambda_2 + \lambda_3 = 40$$

$$\lambda_2 \cdot (\lambda_2 + \lambda_3) = 40 \lambda_2$$

$$\lambda_2^2 + 350 \quad \Rightarrow \quad \lambda_{2,3} = 20 \pm \sqrt{400 - 350}$$

$$= 20 \pm \sqrt{50} = 20 \pm 5\sqrt{2}$$

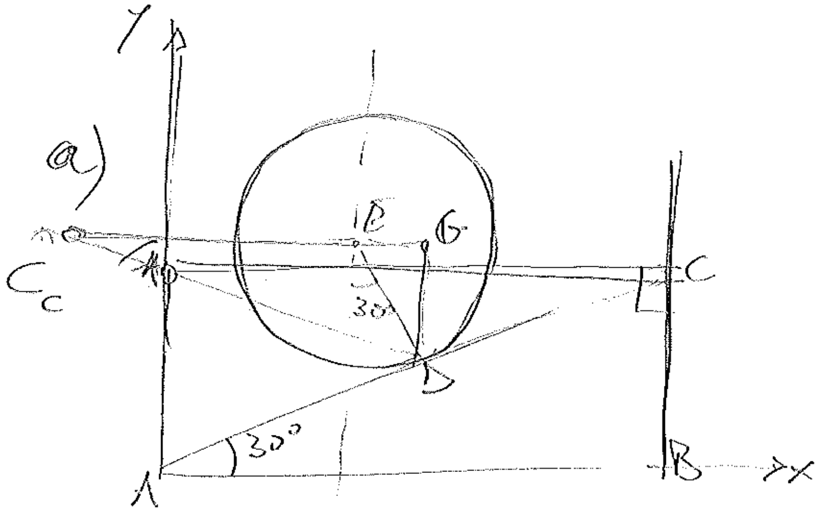


a) determinare centro ist. a $t=0$ $(\vartheta = \vartheta_0)$

b) data v_0 trovare T

c) data $\overline{F}$ trovare $\overline{M}$ per equilibrio.

a)



$$C_A = (0,\ 2l \sin 30) = (0, l)$$

$$C_C = \left(l \cos 30° - GD/\mathrm{tg}\,30°,\ \frac{l}{\cancel{2}} \sin 30 + GD \right)$$

ma $GD = r \cos 30° = \frac{r}{2}\sqrt{3}$

$$C_C = \left(l \frac{\sqrt{3}}{2} - \frac{r}{2}\sqrt{3}/(1/\sqrt{3}),\ \frac{l}{\cancel{2}} \frac{1}{2} + \frac{r}{2}\sqrt{3} \right)$$

$$= \left(l \frac{\sqrt{3}}{2} - \frac{3}{2} r,\ \frac{l}{\cancel{4}\,2} + \frac{r}{2}\sqrt{3} \right)$$

$$\omega_C \overline{C_C D} = v_D = \omega_A \overline{C_A D} \Rightarrow \frac{\omega_C}{\omega_A} = \frac{\overline{C_A D}}{\overline{C_C D}} = \frac{l \sin 30°}{\overline{GD}} = \frac{l}{r\sqrt{3}}$$

$$\omega_A = \frac{v_A}{\overline{C_A A}} = \frac{v_0}{l}$$

$$\omega_C = \frac{v_0}{r\sqrt{3}}$$

$$T = \frac{1}{2}\left(I_\Delta + m\,l^2\right)\omega_x^2 + \frac{1}{2}\left(I_G + m\,\overline{C_cG}^2\right)\omega_c^2$$

$$= \frac{1}{2}\left(\frac{1}{12}m(2l)^2 + m\,l^2\right)\left(\frac{v_0}{l}\right)^2 + \frac{1}{2}\left(\frac{1}{2}m\,r^2 + m\left(\frac{3}{2}r - \frac{r}{2}\right)^2\right)\left(\frac{v_0}{\sqrt{3}\,r}\right)^2$$

$$= \frac{2}{3}m\,v_0^2 + \frac{1}{4}m\,v_0^2 = \frac{11}{12}m\,v_0^2$$

$$F\,v_0\,dt - M\underbrace{\omega_c}_{v_0/r\sqrt{3}}\,dt = 0 \Rightarrow M = \sqrt{3}\,r\,F$$