ESERCIZI DI STATISTICA

Note Title 11/12/2017

$\mathbb{P}_X = N(170, 100)$

$X_1 \quad \mathbb{P}(X_1 > 175) \qquad \mathbb{P}_{X_1} = N(170, 100)$

$Z_1 = \dfrac{X_1 - 170}{10} \qquad \mathbb{P}(X_1 > 175)$

$\mathbb{P}(X_1 > 175) = \mathbb{P}(X_1 - 170 > 175 - 170) =$

$= \mathbb{P}\left(\dfrac{X_1 - 170}{10} > \dfrac{175 - 170}{10}\right) = \mathbb{P}\left(Z_1 > \dfrac{1}{2}\right) =$

$= 1 - \mathbb{P}\left(Z_1 \leq \dfrac{1}{2}\right) = 1 - \Phi(0.5) \simeq 1 - 0.69 = 0.31.$

$\Phi(0.5) \simeq 0.69$

$X_1, X_2 \_\_\_\_ X_{10} \qquad \mathbb{P}_{X_i} = N(170, 100)$

$\mathbb{P}(\overline{X}_{10} > 175) \qquad \mathbb{P}_{\overline{X}_{10}} = N(170, \underbrace{10}_{\text{varianza}})$

$\overline{Z}_{10} = \dfrac{\overline{X}_{10} - 170}{\sqrt{10}}$

$\mathbb{P}(\overline{X}_{10} > 175) = \mathbb{P}(\overline{X}_{10} - 170 > 175 - 170) =$

$= \mathbb{P}\left(\dfrac{\overline{X}_{10} - 170}{\sqrt{10}} > \dfrac{175 - 170}{\sqrt{10}}\right) = \mathbb{P}\left(\overline{Z}_{10} > \dfrac{5}{\sqrt{10}} \cdot \dfrac{\sqrt{10}}{\sqrt{10}}\right)$

$= \mathbb{P}\left(\overline{Z}_{10} > \dfrac{1}{2}\sqrt{10}\right) = 1 - \Phi\left(\dfrac{1}{2}\sqrt{10}\right) = 1 - 0.943 =$

$= 0.057$

$$\frac{\sqrt{10}}{2} \simeq 1.58 \qquad \Phi(1.58) \simeq 0.943$$

X_1 ——— X_{100} $\qquad \mathbb{P}_{X_i} = N(170, 100)$

$$\mathbb{P}_{\overline{X}_{100}} = N(170, 1)$$

$$\overline{Z}_{100} = \frac{\overline{X}_{100} - 170}{1} = \overline{X}_{100} - 170$$

$$\mathbb{P}(\overline{X}_{100} > 175) = \mathbb{P}(\overline{X}_{100} - 170 > 175 - 170) =$$

$$= \mathbb{P}(\overline{Z}_{100} > 5) = 1 - \Phi(5) \qquad \Phi(5) \simeq 0.999997$$

$$\simeq 0$$

— o —

$[0.24, 0.25]$ intervallo di confidenza per il parametro λ con livello di fiducia 0.99

$$X \qquad \mathbb{P}\left(X > \frac{1}{2}\right) = 1 - \mathbb{P}\left(X \leq \frac{1}{2}\right) = 1 - \left(1 - e^{-\lambda/2}\right) = e^{-\lambda/2}$$

$$0.99 = \mathbb{P}(\lambda \in [0.24, 0.25]) = \mathbb{P}(0.24 < \lambda < 0.25) =$$

$$= \mathbb{P}\left(-\frac{0.25}{2} < -\frac{\lambda}{2} < -\frac{0.24}{2}\right) = \mathbb{P}\left(-0.125 < -\frac{\lambda}{2} < -0.12\right)$$

$$= \mathbb{P}\left(\underbrace{e^{-0.125}}_{0.882} < e^{-\lambda/2} < \underbrace{e^{-0.12}}_{0.887}\right)$$

$1-\alpha$

$$\left(\bar{X} - \frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}},\ \bar{X} + \frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}}\right)$$

Semiampiezza $\frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}}$

8k con la seconda metodica

k con la prima metodica

2^a : $\frac{S_2}{\sqrt{8k}} t_{8k-1,1-\frac{\alpha}{2}} = \frac{S_1\sqrt{5}}{\sqrt{8k}} t_{8k-1,1-\frac{\alpha}{2}}$

1^a : $\frac{S_1}{\sqrt{k}} t_{k-1,1-\frac{\alpha}{2}}$

Preferisco 1^a metodica SSE $\frac{\cancel{S_1}}{\sqrt{\cancel{k}}} t_{k-1,1-\frac{\alpha}{2}} < \frac{\cancel{S_1}\sqrt{5}}{\sqrt{8\cancel{k}}} t_{8k-1,1-\frac{\alpha}{2}}$

SSE $t_{k-1,1-\frac{\alpha}{2}} < \frac{\sqrt{5}}{\sqrt{8}} t_{8k-1,1-\frac{\alpha}{2}}$

— o —

$n = 250$ $\bar{x} = 65$ $s^2 = 300$

$$\left(\bar{X} - \frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}},\ \bar{X} + \frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}}\right)$$

$1-\alpha = \frac{90}{100}$ $\alpha = \frac{1}{10}$ $1-\frac{\alpha}{2} = 0.95$ $n-1 = 249$

$$\frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}} = \frac{\sqrt{300}}{\sqrt{250}} \underbrace{t_{249,0.95}}_{\simeq 1.66} \simeq \sqrt{\frac{6}{5}}\, 1.66 \simeq 1.815$$

$(65 - 1.815,\ 65 + 1.815) = (63.185,\ 66.815)$

intervallo di confidenza al 90% per la spesa media

$$\left(\frac{(n-1)s^2}{\chi^2_{n-1,1-\frac{\alpha}{2}}}, \frac{(n-1)s^2}{\chi^2_{n-1,\frac{\alpha}{2}}}\right) = \left(\frac{249 \cdot 300}{286.81}, \frac{249 \cdot 300}{213.47}\right)$$

$\chi^2_{n-1,1-\frac{\alpha}{2}} = \chi^2_{249,\ 0.95}$ $\quad$ $\chi^2_{n-1,\frac{\alpha}{2}} = \chi^2_{249,\ 0.05}$

$$\simeq (260.45,\ 349.9)$$

$\mu = 1.4$

1.549 1.389 1.225 1.082 1.682 1.165 1.301
1.213 1.364 1.448

$\bar{x} \simeq 1.342$

$$\left(\bar{X} - \frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}},\ \bar{X} + \frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}}\right)$$

$s \simeq 0.184$ $\quad$ $n = 10$ $\quad$ $t_{9,0.975} \simeq 2.262$

$1-\alpha = 0.95$ $\quad$ $\alpha = 0.05$ $\quad$ $\frac{\alpha}{2} = 0.025$ $\quad$ $1-\frac{\alpha}{2} = 0.975$

$$\frac{S}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}} \simeq \frac{0.184}{\sqrt{10}} \cdot 2.262 = 0.1316 \simeq 0.132$$

$$(1.342 - 0.132,\ 1.342 + 0.132) =$$
$$= (1.21,\ 1.474)$$

$H_0 : \mu = \underbrace{1.4}_{\mu_0}$ $\qquad$ $H_A \quad \mu \neq 1.4$

Accetto H_0 se $\bar{x} \in \left(\mu_0 - \frac{s}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}},\ 1.4 + \frac{s}{\sqrt{n}} t_{n-1,1-\frac{\alpha}{2}}\right)$

$\alpha = 0.05$ $\qquad$ $s \simeq 0.184$ $\quad$ $\sqrt{n} = \sqrt{10}$

$t_{n-1,1-\frac{\alpha}{2}} = t_{9,0.975} \simeq 2.262$

Accetto H_0 se $\bar{x} \simeq 1.342 \in (1.4 - 0.132, 1.4 + 0.132)$

Accetto H_0 $1.342 \in (1.268, 1.532)$

$H_0)$ $\mu \leq 1.4$ $H_A: \mu > 1.4$

Accetto H_0 $\bar{x} < \mu_0 + \frac{s}{\sqrt{n}} t_{n-1, 1-\alpha}$

Accetto H_0 se $1.342 < 1.4 + 0.132$ ok

— o —

878 933 824 795 954 769 771 888 802 802

— o —

943 1012 1001 861 1209 883 867 788 890 917

X_1 — X_{10} $N(\mu_x, \sigma^2)$ n

Y_1 — Y_{10} $N(\mu_y, \sigma^2)$ k

$\mathbb{P}_{\bar{X}} = N\left(\mu_x, \frac{\sigma^2}{n}\right)$ $\mathbb{P}_{\bar{Y}} = N\left(\mu_y, \frac{\sigma^2}{k}\right)$

$\mathbb{P}_{\bar{X}-\bar{Y}} = N\left(\mu_x - \mu_y, \frac{\sigma^2}{n} + \frac{\sigma^2}{k}\right) = N\left(\mu_x - \mu_y, \sigma^2\left(\frac{1}{n} + \frac{1}{k}\right)\right)$

$\cancel{\mathbb{P}_{S_x^2}}$ $V_x = \frac{(n-1) S_x^2}{\sigma^2}$ $\mathbb{P}_{V_x} = \chi^2_{n-1}$

$V_y = \frac{(k-1) S_y^2}{\sigma^2}$ $\mathbb{P}_{V_y} = \chi^2_{k-1}$

$\mathbb{P}_{V_x + V_y} = \chi^2_{(k-1)+(n-1)} = \chi^2_{n+k-2}$

$V_x + V_y = \frac{1}{\sigma^2}\left((n-1) S_x^2 + (k-1) S_y^2\right)$

$$Z = \frac{(\bar{X}-\bar{Y}) - (\mu_x - \mu_y)}{\sigma\sqrt{\frac{1}{n}+\frac{1}{k}}}$$

$$\underset{t(n+k-2)}{T} = \frac{Z\sqrt{n+k-2}}{\sqrt{V_x+V_y}} = \frac{(\bar{X}-\bar{Y})-(\mu_x-\mu_y)}{\cancel{\sigma}\sqrt{\frac{1}{n}+\frac{1}{k}}} \frac{\sqrt{n+k-2}}{\cancel{\frac{1}{\sigma}}\sqrt{(n-1)S_x^2+(k-1)S_y^2}}$$

$$1-\alpha = \mathbb{P}\left(|T| < t_{n+k-2,\,1-\frac{\alpha}{2}}\right) =$$

$$= \mathbb{P}\left(\frac{|(\bar{X}-\bar{Y})-(\mu_x-\mu_y)|}{\sqrt{\frac{1}{n}+\frac{1}{k}}} \cdot \frac{\sqrt{n+k-2}}{\sqrt{(n-1)S_x^2+(k-1)S_y^2}} < t_{n+k-2,\,1-\frac{\alpha}{2}}\right)$$

$$= \mathbb{P}\left(|(\bar{X}-\bar{Y})-(\mu_x-\mu_y)| < \underbrace{\frac{\sqrt{\frac{1}{n}+\frac{1}{k}}\sqrt{(n-1)S_x^2+(k-1)S_y^2}}{\sqrt{n+k-2}}\, t_{n+k-2,\,1-\frac{\alpha}{2}}}_{\Theta}\right)$$

$$= \mathbb{P}\left(\mu_x-\mu_y \in \left((\bar{X}-\bar{Y}) - \Theta,\ (\bar{X}-\bar{Y}) + \Theta\right)\right)$$

$k=n$

$$\Theta = \frac{\sqrt{\frac{\cancel{2}}{n}}\sqrt{\cancel{(n-1)}(S_x^2+S_y^2)}}{\cancel{\sqrt{2}}\,\cancel{\sqrt{n-1}}}\, t_{2n-2,\,1-\frac{\alpha}{2}}$$

$\bar{x} = 841.6$ $\quad S_x = 66.95$ $\quad n = 10$ $\quad 2n-2 = 18$

$\bar{y} = 937.1$ $\quad S_y = 116.5$ $\quad 1-\alpha = 0.95$ $\quad 1-\frac{\alpha}{2} = 0.975$

$$(841.6 - 937.1) - \sqrt{\frac{(66.95)^2+(116.5)^2}{10}}\, t_{18,\,0.975}\ ,$$

$$(841.6 - 937.1) + \underbrace{\sqrt{\frac{(66.95)^2+(116.5)^2}{10}}\, \underbrace{t_{18,\,0.975}}_{\simeq 2.101}}_{\simeq 77.36}$$

$$\mu_x - \mu_y \in \left(-95.5 - 77.36 \, , -95.5 + 77.36\right) \quad \text{al } 95\%$$

$$\mu_y - \mu_x \in \left(\underbrace{95.5 - 77.36}_{>0} \, , \; 95.5 + 77.36\right)$$