A new synchronization method and its role in the problem of time

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DICE 2010 Castiglioncello, September 17, 2010

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Motivation

Why do we need a global time?

- Economic or legal reasons: you sign a contract to receive after *one year* a certain sum in exchange of some service. If locations are not specified a global time is needed to make sense of the contract. Or: you make a discovery and claim priority.

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- To make sense of appointments: "Let us meet in one hour at the station".

Poincaré-Einstein synchronization convention

Establishes that the one-way transmission delay equals half the two-way transmission time (independent of the synchronization as measured with one clock)

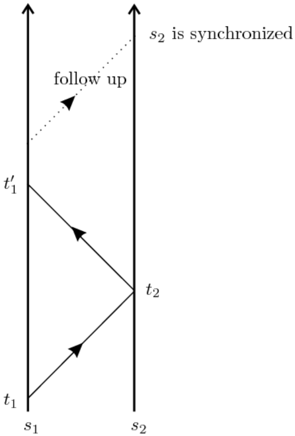
To synchronize reset the second clock so as to replace

$$t_2 \rightarrow \frac{1}{2}(t_1' + t_1)$$

thus

$$t_1' - t_2 = t_2 - t_1$$

namely with it the speed of light is isotropic.



Historical note

The Poincaré-Einstein synchronization convention was currently used by telegraphers in the middle of the XIX century to take into account delays in telegraphic signals running on cables laid under the Atlantic ocean. Accurate time measurements were needed to get longitudes.
Poincaré in 1900 (Leiden lecture)

> *Let us suppose that there are some observers placed at various points, and they synchronize their clocks using light signals. They attempt to adjust the measured transmission time of the signals, but they are not aware of their common motion, and consequently believe that the signals travel equally fast in both directions. They perform observations of crossing signals, one traveling from A to B, followed by another traveling from B to A. The local time t is the time indicated by the clocks which are so adjusted.*

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It is clear that for Poincaré this time, called by him "local time" was fictitious because he assumed the existence of the ether.
He continues in 1904 (St. Louis conference)

> *A sends its signal when its clock marks the hour 0, and the station B perceives it when its clock marks the hour t. The clocks are adjusted if the slowness equal to t represents the duration of the transmission, and to verify it the station B sends in its turn a signal when its clock marks 0; then the station A should perceive it when its clock marks t. The time-pieces are then adjusted.*

Historical note II

...and then Einstein in 1905

> *We have not defined a common "time" for A and B, for the latter cannot be defined at all unless we establish by definition that the "time" required by light to travel from A to B equals the "time" it requires to travel from B to A. Let a ray of light start at the "A time"* t_A *from A towards B, let it at the "B time"* t_B *be reflected at B in the direction of A, and arrive again at A at the "A time"* t'_A. *In accordance with definition the two clocks synchronize if* $t_B - t_A = t'_A - t_B$.

Both recognized the theoretical importance of this synchronization when coming to light signals.

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As Peter Galison points out, Poincaré, as a permanent member of the French Bureau of Longitude, was concerned about accurate maps and delay times in telegraphic transoceanic signals. Einstein, while working at the Swiss patent office in Bern, reviewed patents for synchronizing clocks.

Common misconception

The one-way speed of light with its isotropy depends on the synchronization convention adopted, so to say that the speed of light is constant on the frame is devoid of physical content.

Poincaré was a conventionalist and would have shared this view but this statement is wrong.

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We need to learn more on the consistency of synchronization!

Is Einstein's convention consistent?

Does it lead to a time foliation? We need to prove

(a) Clocks once synchronized remain synchronized.

(b1) Reflexivity.

(b2) Symmetry.

(b3) Transitivity.

Unfortunately the approach in almost every textbooks is the same as Einstein's (1905)

> *We assume that this definition of synchronism is free from contradictions, and possible for any number of points; and that the following relations are universally valid ... [he writes (b2) and (b3)]*

In fact some authors consider the consistency of Einstein's synchronization as an assumption of special relativity theory.
This position ignores completely further developments by Silberstein, Weyl, Reichenbach, Macdonald...

Solution of consistency problem

First given by Silberstein (1914) but clearly stated for the first time by Alan Macdonald (1983!).

Theorem

The conditions (a),(b1),(b2) and (b3) hold if and only if the following two experimentally verifiable (as synchronization independent) conditions hold

- z=0 Clocks are syntonized, that is, if two signals are sent from s_1 at times t_1 and t'_1 and they are respectively received at times t_2 and t'_2 st s_2 then

$$t'_1 - t_1 = t'_2 - t_2.$$

- △ (Reichenbach's round-trip condition) The time it takes light to cover a triangular path (through reflections over suitable mirrors) does not depend on the direction taken around the triangle.

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$$t_1' - t_1 = t_2' - t_2.$$

△ (Reichenbach's round-trip condition) The time it takes light to cover a triangular path (through reflections over suitable mirrors) does not depend on the direction taken around the triangle.

Observation

This theorem is not limited to light signals, nor special or general relativity is assumed from the start. The framework is a space S whose points are called clocks and a signal propagating through it.

Solution of consistency problem II

Even if the Einstein's convention can be consistently applied the one-way velocity of light for the so synchronized clocks does not need to be constant over the frame.
We need a stronger assumption.
I proved (2002)

Theorem

If the average speed of light over a *closed* path is a constant c independent of the path then the clocks can be consistently Einstein's synchronized and with respect to the so synchronized clocks the one-way velocity of light is c (hence isotropic).

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Coming to our earlier issue

Thus the physical content of the naive statement "the speed of light is constant over the frame" is "the speed of light is constant over closed paths", and this is what we need for synchronization and what is ultimately checked in experiments.

Weyl vs Reichenbach

Weyl's round-trip condition

In the frame the average speed of light is constant over closed paths (but you need a notion of length).

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Rotating frame and Sagnac effect

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- Yes, if we insist in using Einstein's synchronization convention. Example of the rotating platform.
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Any good method must be *democratic*, that is, it must not privilege any clock so that the time you get is an intrinsic property of the system.

The new method

Define $w(s_1, s_2, s_3)$ as the difference between the time it takes the signal to cover the paths $s_1 s_2 s_3 s_1$ and the opposite path $s_1 s_3 s_2 s_1$. The function w is (twice) the Sagnac holonomy and $w = 0$ iff the Reichenbach's round-trip condition holds.
It is possible to prove that w is skew-symmetric and

$$w(s_2, s_3, s_4) - w(s_3, s_4, s_1) + w(s_4, s_1, s_2) - w(s_1, s_2, s_3) = 0.$$

Let μ be a unit measure on S, the space of clocks, then

$$\delta(s_1, s_2) = \int_S w(s_1, s_2, s) \, \mathrm{d}\mu(s),$$

is a skew-symmetric function such that

$$w(s_1, s_2, s_3) = \delta(s_1, s_2) + \delta(s_2, s_3) + \delta(s_3, s_1).$$

Furthermore, I proved that any skew-symmetric function that satisfies the latter equality leads to a consistent synchronization by setting (previous notation)

$$t_2 = \frac{1}{2}(t_1' + t_1) + \frac{1}{2}\delta(s_1, s_2)$$

The new method...

...in short

The new synchronization method adds a correction to Einstein's which amounts to an average of the Sagnac effect (which is observable). Contrary to Einstein's it is consistent even in absence of round-trip conditions. The new synchronization method reduces to Einstein's for vanishing Sagnac effect (which is the case in which Einstein's convention works).

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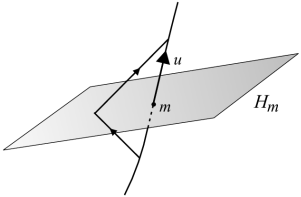
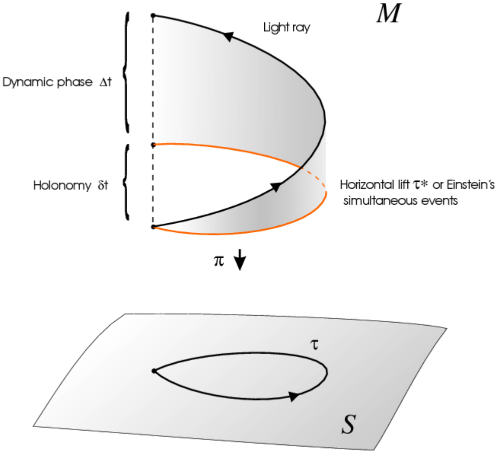
The role of measures

In order to make the average, a finite measure μ is needed, thus we get a time for each syntonized frame provided it admits a natural finite measure.
A relevant case are the conformal stationary frames in general relativity (e.g. Kerr metric), the measure is induced from the space metric of the quotient space.

A local approach in general relativity

The previous method is satisfactory but non-local (because of the average).
Einstein's synchronization is local.
A frame is a congruence of timelike curves. Einstein's convention corresponds to timelike orthogonality.





A different horizontal splitting (last slide)

Frobenious theorem states that the ker of $u_\alpha \mathrm{d}x^\alpha$ is not integrable because the frame has vorticity in general.
We need a different way of splitting the tangent space, namely the horizontal space is the ker of $v_\alpha \mathrm{d}x^\alpha$, where

$$\epsilon^{\delta\alpha\beta\gamma} v_\alpha v_{\beta;\gamma} = 0$$

Furthermore v_α must be constructed from local measurable quantities (acceleration, vorticity, etc.)

Theorem

In a stationary frame, if $v_\alpha = u_\alpha + \psi^m(x) m_\alpha$ where

$$m_\alpha = w_{\alpha\beta} a^\beta \neq 0 \tag{1}$$

$$\psi^m(x) = \frac{a^2 + w^2 - \sqrt{(a^2+w^2)^2 - 4a^2 w^2 \sin^2\theta}}{2a^2 w^2 \sin^2\theta} \tag{2}$$

and θ is the angle between angular velocity and acceleration, then $\epsilon^{\delta\alpha\beta\gamma} v_\alpha v_{\beta;\gamma}$ is proportional to the Riemann tensor and hence vanishes, and is integrable, in the weak field limit.

Conclusions

- I studied the problem of assigning a time function to a frame under minimal assumptions.
- The problem can be solved in many cases of interest as round-trip conditions are no more necessary.
- Establishing what is the time in specific frame examples can be difficult in practice. The method, however, tells us that the time exists a fact that may have implications for canonical quantization schemes.
- Local approaches can also be fruitful but require more investigation.